[Total No. of Printed Pages:3] **CODE NO:- Z-281**

FACULTY OF ENGINEERING

S.E (All Branches) Examination - June – 2015

Engineering Mathematics-IV

(Revised)

[Time: Three *Hours*] **[Max. Marks: 80]**

"Please check whether you have got the right question paper."

N.B
i) Q No.1 and 6 are compulsory.
ii) Solve any two questions from remaining of each section.
iii) Figures to right indicate full marks.
iv) Assume suitable data, if necessary.

SECTION-A

Q.1 Solve any five from the following 10

a) If the real part of an analytic function $f(z) = u(r,\theta) + i\vartheta(r,\theta) is - r\sin\theta$, then find f (z).

b) If $u = \cos ax \sin hy$ is harmonic, then find 'a'.

c) Find the image of the line y =-x under the transformation W= z^2.

d) Evaluate $\int_0^{1+i} z^2 dz$ along y =x.

e) Evaluate $\int_c \frac{1}{z-a} dz$, where C is the circle with center at 'a' and radius is r.

f) Find the poles of the function and the corresponding reduces at each pole of $f(z) = \frac{z+1}{z^2(z-2)}$

g) Solve $2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$

OR

Find the z-transform of $f(k) = \frac{1}{k}, k \geq 1$.

h) Solve $\frac{\partial^2 z}{\partial x \partial y} = x$

OR

Find z-transform of $f(k) = e^{ka}, k \geq 0$

Q.2 a) Test whether the function $f(x) = e^{-x}(\cos y - i\sin y)$ is analytic or not? 05

b) Find the bilinear transformation which maps the points $z = 0, i, \infty$ onto the points$W = 0, \frac{1}{2}, \infty$. 05

c) Solve $\frac{\partial^2 y}{\partial t^2} c^2 \frac{\partial^2 y}{\partial x^2}$, subject to the conditions $y(0,t) = 0, y(2l,t) = 0, \frac{\partial y}{\partial t} = 0\ at\ t = 0$ and 05

$$y(x,o) = \frac{bx}{l}, 0 \leq x \leq l$$

$$\frac{b}{l}(2l - x), l \leq x \leq 2l$$

OR

Find the z-transform of $\frac{2^k}{k}, k \geq 1$.

Q.3 a) Prove that $u = r^3 cos3\theta$ – r sinθ harmonic, and find its conjugate harmonic. 05

b) Evaluate $\int_0^{\pi-\pi i} e^{\bar{z}} dz$, along the curve x=t, y =-t. 05

c) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, with the conditions u(l, t) for all $t \geq 0, \frac{\partial u}{\partial t} = 0$ at x =0 and u(x,0)= 20x for 0<x<l 05

OR

Find the inverse z-transform of $\frac{z^3}{(z-3)(z-2)^2}, |z| > 3$

Q.4 a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ for 0<|z-1|<2. 05

b) Evaluate $\oint_c \frac{\cos hiz}{z^2+4z+3} dx$, where c is |z|=2 by Cauchy's integral formula. 05

c) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions $u(0,y) = u(a,y) = 0$ for $0 \le y \le b, u(x,b) = 0$and $u(x,o) = x(a-x), for\ 0 \le x \le a$ 05

OR

Solve the difference equation by z-transform $Y_{n+2} + 2Y_{n+1} + Y_n = n, Y_0 = Y_1 = 0$

Q.5 a) Show that the map of the real axis of z-plane on the W-plane under the transformation $W = \frac{1}{z+i}$ is a circle and find its radius and center. 05

b) Evaluate $\oint_c \frac{dz}{z^2 \sin hz}$, where c is |z-1|=2 by Cauchy's Residue theorem. 05

c) Evaluate $\int_0^{2\pi} \frac{1}{(5-3\sin\theta)^2} d\theta$,by residues theorem. 05

SECTION -B

Q.6 Solve <u>any five</u> from the following 10

a) Find laplace transform of $\cos t \log t\ \partial(t-\pi)$

b) Find L[f (t)] and L [f^1 (f)] of the function$f(t) = e^{-5t} \sin t$.

c) Find Laplace transform of the function

$$f(t) = e^{t-a}, t > a$$

$$0, t < a$$

d) Find the inverse Laplace transform of $\frac{4s+12}{s^2+8s+16}$

e) Find the inverse Laplace transform of $\frac{s\, e^{-3s}}{s^2-1}$

f) find inverse Laplace transform of $\frac{2+5s}{s^2}$

g) Find the Fourier sine transform of

$f(x) = k, 0 < x < a$

$= 0, x < 0.$

h) Find the Fournier transform of

$f(x) = x\, e^{-x}, x > 0$

$= 0, x < 0.$

Q.7 a) Find the Laplace transform of$e^{-3t} \int_0^t t \sin 3t\ dt$. 05

b) Find the inverse Laplace transform of $2 \tan h^{-1} s$. 05

c) Using Fournier transform, solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, for\ 0 \le x < \infty, t > 0$ given the conditions 05

i) $u(x,0) = 0\ for\ x \ge 0$ ii) $\frac{\partial u(0,t)}{\partial x} = -a(constant)$ iii) $u(x,t) is\ bounded$

Q.8 a) Evaluate $\int_0^\infty \frac{e^{-t}(1-\cos 2t)}{2t} dt$ 05

b) Find the inverse Laplace transform of $\frac{1}{(s-3)(s+3)^2}$ by convolution theorem. 05

c) Solve the integral equation$\int_0^\infty f(x) cos\lambda x\ dx = e^{-\lambda}, \lambda \ge 0$. 05

Q.9 a) Express the following function in terms of Heaviside unit step function and hence find their Laplace transform 05

$f(x) = t, 0 < t < 2$

$t^2, t > 2$

b) Solve $y'' + 4y = \partial(t), y(0) = 0, y'(0) = 0$ by Laplace transform. 05

c) Find the Fourier cosine transform of 05

$f(x) = x, 0 < x < \frac{1}{2}$

$= 1 - x, \frac{1}{2} < x < 1$

$= 0, x > 1$

Q.10 a) Find the Laplace transform of 05

$f(t) = \cos wt\ , 0 < t < \pi/w$

$0, \pi/w < \ t < 2\pi/w$ and $f(t) = f(t + \frac{2\pi}{w})$

b) Find the Fourier transform of 05

$f(x) = 1 + \frac{x}{a}, -a < x < 0$

$= 1 - \frac{x}{a}, 0 < x < a$

$= 0$, otherwise

c) Solve $\frac{dx}{dt} = y + e^t, \frac{dy}{at} = \sin t - x, x(0) = 1, y(0) = 0$ by Laplace transform. 05